

## 例 42 求函数的极值

1 已知函数  $f(x) = \sin x \cdot \tan x$ .

(1) 求  $g(x) = f(x) + 3\cos x$  在  $x \in \left(0, \frac{\pi}{2}\right)$  上的极值.

(2) 求  $x \in \left[0, \frac{\pi}{2}\right)$  上  $f(x) \geq x^2$ .

解 (1)

(2)

解

解

1 求函数的极值

2 求函数的极值  $h(x) = f(x) - x^2 = \sin x \cdot \tan x - x^2$  在  $x \in \left(0, \frac{\pi}{2}\right)$  上的极值.

(1)

解  $g(x) = \sin x \cdot \tan x + 3\cos x$  求  $g'(x) = \frac{-\sin x \cdot \cos 2x}{\cos^2 x}$

在  $x \in \left(0, \frac{\pi}{2}\right)$  上  $\sin x > 0$

在  $g'(x) > 0$  时  $\cos 2x < 0$  在  $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

在  $g'(x) < 0$  时  $\cos 2x > 0$  在  $x \in \left(0, \frac{\pi}{4}\right)$

在  $g(x)$  在  $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$  上单调递增在  $\left(0, \frac{\pi}{4}\right)$  上单调递减

在  $g(x)$  在  $\left(\frac{\pi}{4}\right) = 2\sqrt{2}$

(2)

$$\boxed{\phantom{000}} \quad f(x) \geq x^2 \quad \boxed{\phantom{000}} \quad f(x) - x^2 \geq 0 \quad \boxed{\phantom{000}}$$

$$h(x) = f(x) - x^2 = \sin x \cdot \tan x - x^2, x \in \left[0, \frac{\pi}{2}\right) \therefore h'(x) = \sin x \left( \frac{1}{\cos^2 x} + 1 \right) - 2x \geq \sin x \cdot \frac{2 \cos x}{\cos^2 x} - 2x$$

$$= \frac{2 \sin x}{\cos x} - 2x = 2(\tan x - x)$$

$$\square \quad k(x) = \tan x - x \quad x \in \left[0, \frac{\pi}{2}\right) \quad \square \quad \therefore k'(x) = \frac{1}{\cos^2 x} - 1 \quad \square$$

$$0 < \cos^2 x \leq 1 \Rightarrow \frac{1}{\cos^2 x} \geq 1 \Rightarrow K(x) \geq 0$$

$$K(x) \in \left[0, \frac{\pi}{2}\right) \implies K(0) = 0, K(x) \geq 0, H(x) \geq 0$$

$$\langle H(x) | \left[ 0, \frac{\pi}{2} \right] | H(0) \rangle = 0$$

$$\therefore H(x) \geq 0 \quad \square \square \square \square \square \square \square \square.$$

$$2 \square \square \square \begin{matrix} f(x) = \sin x \\ g(x) = \ln x \\ h(x) = x^2 - ax - 1 \end{matrix} \square$$

$$\forall x \in [0,1] \quad f(x) \geq g(x+1)$$

$$\exists x \in [0,1] \quad e^{f(x)} + h(x) - g(x) > 0 \quad a$$

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$x_0 \in (0,1)$     $F(x_0) = 0$     $F'(x) > 0$     $(0,1)$     $F'(x)$

$$F(x) \geq 0 \quad \square\square\square\square\square\square.$$





$$\square k > 0 \square\square\square g'(x) = 0 \square\square x = \ln 2k \square$$

$$\square \begin{matrix} g'(x) > 0 & x > \ln 2k \\ \square & \square \end{matrix} \therefore g(x) \square\square\square\square\square$$

$$\square g'(x) < 0 \square\square x < \ln 2k \square \therefore g(x) \square\square\square\square\square$$

$$\therefore g(x) \square\square\square\square\square \ln 2k \square\square\square\square\square\square$$

$$\square\square\square\square k, 0 \square\square g(x) \square\square\square\square\square$$

$$\square \begin{matrix} k > 0 & g(x) \\ \square & \square \end{matrix} \square\square\square\square\square \ln 2k \square\square\square\square\square\square$$

$$\square 2 \square \textcircled{1} \square\square\square\square\square k = \frac{1}{2} \square\square\square G(x) = e^x - \frac{1}{2} x^2 - x - 1(x, 0) \square$$

$$G(x) = e^x - x - 1 \square$$

$$\square \begin{matrix} G'(x) = e^x - 1, 0 \\ \square \end{matrix} \square\square G(x) \square [0, +\infty) \square\square\square\square\square\square$$

$$\square\square \begin{matrix} x, 0 \\ \square \end{matrix} \square\square G(x) \dots G(0) = 0 \square\square G(x) \square [0, +\infty) \square\square\square\square\square\square$$

$$\square\square x, 0 \square\square G(x) \dots G(0) = 0 \square$$

$$\square\square \begin{matrix} x, 0 \\ \square \end{matrix} \square\square f(x) \dots x + 1 \square\square\square\square$$

$$\textcircled{2} \square \begin{matrix} h(x) = e^x - kx^2 - 2x - 1 + \sin x(x, 0) \\ \square \end{matrix} \square$$

$$\square \begin{matrix} h(x) \dots 0 \\ \square \end{matrix} \square\square h(0) = 0 \square$$

$$\square \begin{matrix} h(x) = e^x - 2kx^2 - 2 + \cos x(x, 0) \\ \square \end{matrix} \square\square h(0) = 0 \square$$

$$h'(x) = e^x - 2k - \sin x \square h'(0) = 1 - 2k \square$$

$$\therefore h''(x) = e^x - \cos x, 0 \square\square h'(x) \square [0, +\infty) \square\square\square\square\square\square$$

$$\square k, \frac{1}{2} \square\square h'(0) = 1 - 2k, 0 \square\square\square h'(x) \square [0, +\infty) \square\square\square\square\square\square$$

$$\square\square \begin{matrix} x, 0 \\ \square \end{matrix} \square\square h'(x) \dots h'(0) \dots 0 \square\square h(x) \square [0, +\infty) \square\square\square\square\square\square$$



1  $f'(x) \leq f(x)$   $\Leftrightarrow f'(x) \leq 0 \Leftrightarrow a \geq \frac{\ln x + 1}{e^x} \Leftrightarrow F(x) = \frac{\ln x + 1}{e^x} \leq f(x)$

2  $a = 1$   $x \ln x < e^x + \cos x - 1$   $0 < x \leq 1$   $x \ln x \leq 0$   $e^x + \cos x - 1 > 0$   $x > 1$

$$G(x) = e^x + \cos x - x \ln x - 1 \quad (x > 1)$$

$G(x)$   $(1, +\infty)$

1  $f(x) = x \ln x - ae^x \quad (x > 0)$   $f(x) = \ln x + 1 - ae^x$

$f(x) \leq 0 \Leftrightarrow a \geq \frac{\ln x + 1}{e^x}$

$$F(x) = \frac{\ln x + 1}{e^x} \quad F'(x) = \frac{\frac{1}{x} - \ln x - 1}{e^x}$$

$F(1) = 0$   $y = \frac{1}{x} - \ln x - 1$   $(0, +\infty)$   $e^x > 0$

$F(x) = \frac{\ln x + 1}{e^x}$   $(0, 1)$   $(1, +\infty)$

$$F(x) \leq F(1) = \frac{1}{e}$$

$a \geq \frac{1}{e}$   $f(x) \leq 0$

$a \in \left[ \frac{1}{e}, +\infty \right)$

2  $a = 1$   $f(x) = x \ln x - e^x$   $h(x) = g'(x) = \cos x - 1$

$f(x) < h(x)$   $x \ln x < e^x + \cos x - 1$

$0 < x \leq 1$   $x \ln x \leq 0$   $e^x + \cos x - 1 > 1 + \cos 1 - 1 > 0$

$x \ln x < e^x + \cos x - 1$





$$x^2 + x \ln x + x > x(2 + \ln x) - 2(1 - \sin x) \quad x^2 - x + 2 - 2 \sin x > 0 \quad g(x) = x^2 - x + 2 - 2 \sin x$$

证明

$$f(x) = xe^x - x(x + \ln x) = e^{\ln x} x - x(x + \ln x) \quad t = x + \ln x \quad t \in R$$

$$f(x) \geq 1 \quad e^t - at - 1 \geq 0 \quad t \in R \quad h(t) = e^t - at - 1$$

$$a < 0 \quad h\left(\frac{1}{a}\right) = e^{\frac{1}{a}} - 2 < e^0 - 2 < 0 \quad h(t) \geq 0$$

$$a = 0 \quad h(t) = e^t - 1 \quad h(-1) = e^{-1} - 1 = \frac{1}{e} - 1 < 0 \quad h(t) \geq 0$$

$$a > 0 \quad h(t) = e^t - at \quad h(t) \quad (-\infty, \ln a) \quad (\ln a, +\infty)$$

$$\therefore h(t) \quad h(\ln a) = a - a \ln a - 1$$

$$\varphi(a) = a - a \ln a - 1 \quad \varphi'(a) = -\ln a \quad \varphi(a) \quad (0, 1) \quad (1, +\infty)$$

$$\therefore \varphi(a)_{\max} = \varphi(1) = 0 \quad \varphi(a) = a - a \ln a - 1 \geq 0 \quad a = 1$$

$$a \geq 1$$

$$a = 1 \quad xe^x - x - \ln x \geq 1 \quad xe^x \geq x + \ln x + 1$$

$$\therefore x^2 e^x \geq x^2 + x \ln x + x$$

$$x^2 + x \ln x + x > x(2 + \ln x) - 2(1 - \sin x) \quad x^2 - x + 2 - 2 \sin x > 0$$

$$g(x) = x^2 - x + 2 - 2 \sin x \quad g'(x) = 2x - 1 - 2 \cos x$$

$$0 < x < 1 \quad g'(x) \quad g'(1) = 1 - 2 \cos 1 < 1 - 2 \cos \frac{\pi}{3} = 0$$

$$\therefore g(x) \quad (0, 1] \quad g(x) \geq g(1) = 2 - 2 \sin 1 > 0$$

$$x > 1 \quad x^2 - x + 2 - 2 \sin x \geq 0 \quad g(x) > 0$$

$$x \in (0, +\infty) \quad g(x) > 0 \quad x^2 + x \ln x + x > x(2 + \ln x) - 2(1 - \sin x)$$





1  $f(x) = \frac{-x \cos x + \sin x - \pi}{x^2}$   $g(x) = -x \cos x + \sin x - \pi$   $g'(x) = 0$   $(0, 2\pi)$

2  $x \in (0, \pi)$   $0 < \frac{\sin x}{x} < 1$   $0 < a < \pi$   $\frac{\pi}{x} + a \ln x > 1$

1  $f(x) = \frac{-x \cos x + \sin x - \pi}{x^2}$

$g(x) = -x \cos x + \sin x - \pi$   $g'(x) = x \sin x$

$x \in (0, \pi)$   $g'(x) > 0$   $x \in (\pi, 2\pi)$   $g'(x) < 0$

$[g(x)]_{\max} = g(\pi) = 0$   $g(x) \leq 0$   $(0, 2\pi)$   $f(x) \geq 0$

$f(x) \geq 0$   $(0, 2\pi)$

2  $f(x) > a \ln \frac{1}{x} + \frac{\pi}{x} + a \ln x - \frac{\sin x}{x} > 0$

$x \in (0, \pi)$   $0 < \frac{\sin x}{x} < 1$   $0 < a < \pi$   $\frac{\pi}{x} + a \ln x > 1$

$h(x) = x - \sin x$   $h'(x) = 1 - \cos x > 0$   $h(x) = x - \sin x$   $(0, \pi)$

$h(x) > h(0) = 0$   $x - \sin x > 0$

$x > 0, \sin x > 0$   $0 < \frac{\sin x}{x} < 1$

$\varphi(x) = \frac{\pi}{x} + a \ln x$   $\varphi'(x) = -\frac{\pi}{x^2} + \frac{a}{x} = \frac{ax - \pi}{x^2}$

$\varphi'(x) = 0$   $\varphi(x)$   $x_0 = \frac{\pi}{a}$   $x_0 = \frac{\pi}{a} \in (0, \pi)$   $1 < a < \pi$

$\varphi(x_0) = \frac{\pi}{x_0} + a \ln x_0 = a + a \ln \frac{\pi}{a} > a > 1$   $\varphi(x) = \frac{\pi}{x} + a \ln x > 1$

$x_0 = \frac{\pi}{a} \dots \pi$   $0 < a, 1$   $\varphi(x)$   $(0, \pi)$   $\varphi(x) > \varphi(\pi) = 1 + a \ln \pi > 1$

$0 < a < \pi$   $\frac{\pi}{x} + a \ln x > 1$





$$2) \quad g'(x) = e^x - kx - 1 \quad g'(x) = e^x - k$$

$$x \geq 0 \quad e^x \geq 1$$

$$k \leq 1 \quad g'(x) \geq 0 \quad g(x) \uparrow (0, +\infty)$$

$$g(x) \geq g(0) = 0$$

$$k > 1 \quad g'(x) = e^x - e^{\ln k} \quad 0 < x < \ln k \quad g'(x) < 0 \quad g(x) \downarrow (0, \ln k)$$

$$0 < x < \ln k \quad g(x) < g(0) = 0 \quad \text{“} x \geq 0 \quad g(x) \geq 0 \text{”}$$

$$k \in (-\infty, 1]$$

$$3) \quad f'(x) = ae^{ax-1} \cdot \cos x - e^{ax-1} \cdot \sin x = e^{ax-1} (a \cos x - \sin x)$$

$$f'(x) = 0 \quad a \cos x - \sin x = 0 \quad x \in \left(0, \frac{\pi}{2}\right) \quad \tan x = a$$

$$a > 0 \quad x_0 \in \left(0, \frac{\pi}{2}\right) \quad \tan x_0 = a$$

$$x \in (0, x_0) \quad f'(x) > 0 \quad f(x) \uparrow (0, x_0)$$

$$x \in \left(x_0, \frac{\pi}{2}\right) \quad f'(x) < 0 \quad f(x) \downarrow \left(x_0, \frac{\pi}{2}\right)$$

$$x = x_0 \quad f(x) \quad f(x_0) = e^{ax_0-1} \cdot \cos x_0$$

$$f(x_0) > e^{-\frac{1}{a}}$$

$$0 \leq x \leq \frac{\pi}{2} \quad 1 \leq x \leq \sin x \quad 2) \quad e^{x-1} \geq x$$

$$e^{ax_0-1} \geq ax_0 \geq a \sin x_0 \quad f(x_0) = e^{ax_0-1} \cdot \cos x_0 \geq a \sin x_0 \cos x_0 = \frac{a^2}{1+a^2}$$

$$\frac{a^2}{1+a^2} > e^{-\frac{1}{a}} \quad t = -\frac{1}{a} \quad t < 0$$

$$\frac{1}{1+t^2} > e^t \quad (t < 0) \quad (1+t^2)e^t - 1 < 0 \quad (t < 0)$$

$$\varphi(t) = (1+t^2)e^t - 1 \quad (t < 0) \quad \varphi'(t) = (1+t^2)e^t \geq 0$$

$$\varphi(t) \quad (-\infty, 0)$$

$$t < 0 \quad \varphi(t) < \varphi(0) = 0 \quad (1+t^2)e^t - 1 < 0 \quad (t < 0)$$

$$f(x_0) > e^{-\frac{1}{a}}$$

10  $x$

$$1 - \cos x - \frac{1}{2}x^2$$

$$e^x - 1 > x + ax^2 \quad a$$

$$2e^x + \cos x > \sqrt{\ln\left(x + \frac{3}{2}\right)} + \sin x + 2$$

$$1 - \cos x > 1 - \frac{1}{2}x^2 \quad \left(-\infty, \frac{1}{2}\right]$$

$$f(x) = \cos x - \left(1 - \frac{1}{2}x^2\right) \quad f(x) \geq 0$$

$$f(x) = e^x - 1 - x - ax^2 \quad a$$

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$$\square 1 \square \square \quad f(x) = \cos x - \left(1 - \frac{1}{2}x^2\right)$$

$$\therefore f'(x) = -\sin x + x \quad f'(x) = -\cos x + 1 \stackrel{0}{>} 0 \quad x > 0 \quad \square \square \square \square$$

$$\therefore f(x) \quad (0, +\infty) \quad \square \square \square \square \square \quad f(0) = 0 \quad \square$$

$$\therefore f(x) > 0 \quad (0, +\infty) \quad \square \square \square \square$$

$$\therefore f(x) \quad (0, +\infty) \quad \square \square \square \square \square \quad f(0) = 0 \quad \square$$

$$\therefore f(x) > 0 \quad (0, +\infty) \quad \square \square \square \square$$

$$\therefore \cos x > 1 - \frac{1}{2}x^2.$$

$$\square 2 \square \square \quad f(x) = e^x - 1 - x - ax^2 \quad \therefore e^x - 1 - x - ax^2 > 0 \quad (0, +\infty) \quad \square \square \square \square$$

$$\therefore f'(x) = e^x - 1 - 2ax \quad f'(x) = e^x - 2a \quad \square$$

$$\square 2a \leq 1 \quad \square \square \quad a \leq \frac{1}{2} \quad \square \square \quad f(x) > 0 \quad \square$$

$$\therefore f(x) \quad \square \square \square \square \square \square$$

$$\therefore f(x) > 0 \quad (0, +\infty) \quad \square \square \square \square \square \quad f(0) = 0 \quad \square$$

$$\therefore f(x) \quad (0, +\infty) \quad \square \square \square \square \square \quad f(0) = 0 \quad \square$$

$$\therefore f(x) > 0 \quad (0, +\infty) \quad \square \square \square \square \quad \therefore a \leq \frac{1}{2}.$$

$$\square 2a > 1 \quad \square \square \square \quad a > \frac{1}{2} \quad \square$$

$$\therefore f(x) > 0 \Rightarrow x > \ln 2a \quad f(x) < 0 \Rightarrow 0 < x < \ln 2a \quad \square$$

$$\therefore f(x) \quad (0, \ln 2a) \quad \square \square \square \square \square \square \quad f(0) = 0 \quad \square$$

$$\therefore f(x) < 0 \quad (0, \ln 2a)$$

$$\therefore f(x) \quad (0, \ln 2a) \quad f(0) = 0$$

$$\therefore f(x) < 0 \quad (0, \ln 2a)$$

$$\therefore a > \frac{1}{2}$$

$$a \leq \frac{1}{2}$$

$$a = \frac{1}{2} \quad e^x > 1 + x + \frac{x^2}{2}$$

$$e^x + \cos x > e^x + 1 - \frac{1}{2}x^2 > x + 2 > \sin x + 2$$

$$e^x > \sqrt{e} \ln \left( x + \frac{3}{2} \right) \Leftrightarrow e^{x \frac{1}{2}} > \ln \left( x + \frac{3}{2} \right)$$

$$u(x) = e^x - x - 1 \quad u'(x) = e^x - 1$$

$$u'(x) > 0 \Rightarrow x > 0, u'(x) < 0 \Rightarrow x < 0$$

$$u(x) \quad (0, +\infty) \quad (-\infty, 0) \quad u(0) = 0$$

$$u(x) \geq 0 \quad \therefore e^x \geq x + 1$$

$$v(x) = \ln x - x + 1 \quad v'(x) = \frac{1}{x} - 1$$

$$v'(x) > 0 \Rightarrow 0 < x < 1, v'(x) < 0 \Rightarrow x > 1$$

$$v(x) = \ln x - x + 1 \quad (0, 1) \quad (1, +\infty) \quad v(1) = 0$$

$$v(x) \leq 0 \quad \ln x \leq x - 1$$



$$f(x) \text{ 在 } (0, +\infty) \text{ 上单调递增, 在 } (-\infty, 0) \text{ 上单调递减}$$

$$x = \frac{\pi}{2} \quad h\left(\frac{\pi}{2}\right) = ae^{\frac{\pi}{2}} + 2e^{-\frac{\pi}{2}} + (a-2)\frac{\pi}{2} \geq 0$$

$$\left(e^{\frac{\pi}{2}} + \frac{\pi}{2}\right)a \geq \pi - \frac{2}{e^{\frac{\pi}{2}}} > 0 \quad a > 0$$

$$g(x) = h(x) - (a+2)\cos x$$

$$= ae^x + 2e^{-x} + (a-2)x - (a+2)\cos x$$

$$g'(x) = ae^x - 2e^{-x} + (a-2) + (a+2)\sin x$$

$$= \frac{ae^{2x} - 2}{e^x} + (a-2) + (a+2)\sin x$$

$$a \geq 2 \quad x \in [0, \pi] \quad g'(x) \geq 0$$

$$g(x) \text{ 在 } [0, \pi] \text{ 上单调递增}$$

$$x \in (\pi, +\infty)$$

$$g'(x) = ae^x - 2e^{-x} + (a-2) + (a+2)\sin x$$

$$\geq ae^x - 2e^{-x} + (a-2) - (a+2)$$

$$> ae^x - 2e^{-x} - 4 > 4a - \frac{2}{4} - 4$$

$$x \in [0, +\infty) \quad g'(x) \geq 0$$

$$g(x) \geq g(0) = 0$$

$$0 < a < 2 \quad g'(0) = 2(a-2) < 0$$

$$g'(x) = ae^x - 2e^{-x} + (a-2) + (a+2)\sin x$$

$$\geq ae^x - 2e^{-x} + (a-2) - (a+2)$$

$$= ae^x - 2e^{-x} - 4$$



$$F(x) = \sin x - \frac{\sqrt{2}}{2}x \quad F(x) = \cos x - \frac{\sqrt{2}}{2}x$$

$\left(0, \frac{\pi}{4}\right)$ 
 $\square\square F'(x)\square0\square F(x)\square\left[0, \frac{\pi}{4}\right]$ 
 $\square\square\square\square\square\square$

$$\left(\frac{\pi}{4}\right) \leq F'(x) \leq F(x) \leq \left(\frac{\pi}{4}\right) \implies F(0) \leq F(1) \implies \int_0^1 F(x) dx \geq \frac{\sqrt{2}}{2}.$$

$$0 \leq \sin x \leq x \leq 1 \leq H'(x) \leq \cos x \leq 1 \leq 0$$

$\square [0 \pi] : H(x) \leq H(0) - \int_0^x \sin t dt = x$

$$\frac{2}{x} \leq \sin x \leq x \quad x \in [0, 1]$$

$$[x \in [0, 1]] \cdot ax + x^2 + \frac{x^3}{2} + 2(x+2)\cos x - 4$$

$$2)x + x^2 + \frac{x^3}{2} - 4(x+2)\sin^2 \frac{x}{2} \leq (a+2)x + x^2 + \frac{x^3}{2} - 4(x+2)\left(\frac{\sqrt{2}}{4}x\right)^2 = (a+2)x.$$

2.  $ax^2 + x^2 + \frac{x^3}{2} + 2(x+2)\cos x \leq 4 \quad \forall x \in [0, 1]$

$$ax + x^2 + \frac{x^2}{2} + 2(x+2)\cos x \leq 4 \quad x \in [0, 1]$$

$$X^2 + \frac{X^3}{2} + 2(X+2)\cos X - 4 = (a+2)X + X^2 + \frac{X^3}{2} - 4(X+2)\sin^2 \frac{X}{2}$$

$$2) X + X^2 + \frac{X^3}{2} - 4(X+2) \left( \frac{X}{2} \right)^2$$

$$2) X - X^2 - \frac{X^3}{2} \geq (a+2)X - \frac{3}{2}X^2 = -\frac{3}{2}X \left[ X - \frac{2}{3}(a+2) \right].$$

$$s \in (0, 1) \implies x_0 \leq \frac{a+2}{3} \leq \frac{1}{2} \implies \text{converges}$$

$$x + \frac{x^2}{2} + 2(x_0 + 2) \cos x_0 - 4 > 0 \quad \text{a} \quad 2$$

$$ax + x^2 + \frac{x^3}{2} + 2(x+2)\cos x - 4 \leq 0 \quad x \in [0, 1]$$

$$a \in (-\infty, 2]$$

$$1 \leq x \leq 2$$

$$a \leq f(x)$$

$$a \leq f(x) \quad a \leq f(x)_{\min}$$

$$a \geq f(x) \quad a \geq f(x)_{\max} \quad f(x)_{\min} \geq 0 \quad f(x)_{\max} \leq 0 \quad a$$

$$a \geq f(x)$$

$$x \in [0, 1] \quad \frac{\sqrt{2}}{2} x \leq \sin x \leq x$$

$$ax + x^2 + \frac{x^3}{2} + 2(x+2)\cos x \leq 4 \quad x \in [0, 1]$$

$$x \in (-\infty, -2]$$

$$a \leq f(x)$$

$$a \leq f(x)$$

$$h(x) = \sin x - \frac{\sqrt{2}}{2} x, h(x) = \cos x - \frac{\sqrt{2}}{2} x$$

$$x \in \left[0, \frac{\pi}{4}\right] \quad h(x) \geq 0 \quad h(x) \quad x \in \left[\frac{\pi}{4}, 1\right] \quad h(x) \leq 0 \quad h(x)$$

$$h(1) > 0, \quad x \in [0, 1] \quad h(x) \geq 0, \quad \sin x \geq \frac{\sqrt{2}}{2} x$$

$$u(x) = \sin x - x, u(x) = \cos x - 1 \leq 0 \quad u(x) \quad u(x) \leq u(0) = 0 \quad \sin x \leq x$$

$$\frac{\sqrt{2}}{2} x \leq \sin x \leq x, x \in [0, 1]$$

$$x \in [0, 1] \quad$$

$$ax + x^2 + \frac{x^3}{2} + 2(x+2)\cos x - 4 = (a+2)x + x^2 + \frac{x^3}{2} - 4(x+2)\sin^2 \frac{x}{2}$$





$$\square\square x\ln x\geq x-1>\frac{3\sin x}{2+\cos x}-1\square\square\square 2x+x\cos x-3\sin x>0(x>0)\square\square\square\square\square x\geq\pi\square\square 0<x<\pi\square\square\square\square\square\square\square\square\square\square$$

$\square\square\square\square$

$$\square 1\square\square\square\square\square g(x)=x\ln x-\frac{x}{a}x+1(x>0)\square\square\square g'(x)=\ln x+1-\frac{1}{a}(x>0).$$

$$\square g'(x)>0\square\square x>e^{a-1}\square\square g'(x)<0\square\square 0<x<e^{a-1}.\square\square g(x)\square(0,e^{a-1})\square\square\square\square\square\square\square(e^{a-1},+\infty)\square\square\square\square\square\square$$

$$g(x)_{\min}=g(e^{a-1})=1-\frac{1}{e^{a-1}}.$$

$$\square a=1\square\square g(e^{a-1})=1-\frac{1}{e^{a-1}}=0\square\square g(x)\square\square\square\square\square\square\square\square x=1\square$$

$$\square a<1\square\square g(e^{a-1})=1-\frac{1}{e^{a-1}}>0\square\square g(x)\square\square\square\square\square\square$$

$$\square a>1\square\square g(e^{a-1})=1-\frac{1}{e^{a-1}}<0\square\square g(e^a)=-\frac{1}{e^a}-\frac{1}{e^a}+1=\frac{e^a-2}{e^a}>0\square\square\square g(x)\square(e^{a-1},+\infty)\square\square\square\square\square\square\square\square$$

$$g(e^a)=-\frac{1}{e^a}-\frac{1}{e^a}+1=\frac{e^a-2}{e^a}>\frac{e^a-2}{e^a}>0\square\square\square g(x)\square(0,e^{a-1})\square\square\square\square\square\square\square.$$

$$\square\square\square\square\square\square a=1\square\square g(x)\square\square\square\square\square\square\square\square\square\square a<1\square\square g(x)\square\square\square\square\square\square a>1\square\square g(x)\square\square\square\square\square\square.$$

$$\square 2\square\square\square\square\square\square 1\square\square\square\square\square a=1\square\square g(x)\geq g(x)_{\min}=0\square\square x\ln x\geq x-1.$$

$$\square\square x\ln x>\frac{3\sin x-\cos x-2}{2+\cos x}\square\square\square x\ln x\geq x-1>\frac{3\sin x}{2+\cos x}-1\square$$

$$\square\square x>\frac{3\sin x}{2+\cos x}(x>0)\square\square\square 2x+x\cos x-3\sin x>0(x>0).$$

$$\square h(x)=2x+x\cos x-3\sin x(x>0).$$

$$\square x\geq\pi\square\square h(x)=2x+x\cos x-3\sin x=x(1+\cos x)+x-3\sin x>x-3>0.$$

$$\square 0<x<\pi\square\square h(x)=2-2\cos x-x\sin x\square\square f(x)=h(x)\square\square f'(x)=\sin x-x\cos x.$$

$$\square\square\varphi(x)=f(x)\square\square\varphi'(x)=x\sin x>0\square$$

$f'(x) \in (0, \pi)$   $f'(x) = \sin x - x \cos x > f'(0) = 0$

$f(x) \in (0, \pi)$   $f(x) = h(x) > h(0) = 0$

$h(x) \in (0, \pi)$   $h(x) > h(0) = 0$

$f(x) > \frac{3\sin x - \cos x - 2}{2 + \cos x}$

证明

证明函数在区间上单调递增或递减的方法

① 证明函数在区间上单调递增

② 证明函数在区间上单调递减

③ 证明函数在区间上单调递增或递减

证明函数在区间上单调递增或递减的方法

$f(x) = ax - \sin x, x \in (0, +\infty) (a \in \mathbb{R})$

(1)  $f(x) > 0$   $a$  的取值范围

(2)  $a = 1$   $2f(x) + \cos x > e^x$

证明(1)  $[1, +\infty)$  (2) 证明

证明

证明

(1) 证明  $f(x)$  在区间上单调递增或递减  $a$  的取值范围

(2) 证明函数在区间上单调递增或递减  $(2x - 2\sin x + \cos x)e^x > 1$

证明

(1)  $f(x) = a - \cos x$

$a \geq 1$   $f(x) \geq 0$   $f(x) \in (0, +\infty)$   $f(x) > f(0) = 0$

$a \leq -1$   $f(x) \leq 0$   $f(x) \in (0, +\infty)$   $f(x) < f(0) = 0$

$$-1 < a < 1 \quad f(x) = 0 \quad (0, \pi) \quad x_0 \quad \cos x_0 = a$$

$$0 < x < x_0 \quad f(x) < 0 \quad f(x) \quad (0, x_0) \quad f(x) < f(0) = 0$$

$$a \quad [1, +\infty)$$

$$(2) \quad a = 1 \quad f(x) = x - \sin x$$

$$2f(x) + \cos x > e^x \quad 2x - 2\sin x + \cos x > e^x \quad (2x - 2\sin x + \cos x)e^x > 1$$

$$g(x) = (2x - 2\sin x + \cos x)e^x$$

$$g'(x) = [(2 - 2\cos x - \sin x) + (2x - 2\sin x + \cos x)]e^x$$

$$= [2(x - \sin x) + 2 - \sqrt{2}\sin(x + \frac{\pi}{4})]e^x$$

$$(1) \quad x > \sin x \quad 2 - \sqrt{2}\sin(x + \frac{\pi}{4}) \geq 2 - \sqrt{2} > 0$$

$$h(x) > 0 \quad g(x) \quad (0, +\infty) \quad g(x) > g(0) = 1$$

$$\forall x \in (0, +\infty) \quad a = 1 \quad 2f(x) + \cos x > e^x$$

(1) .

(2) .

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